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LETTER TO THE EDITOR

Driving slow-light solitons by a controlling laser field

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Abstract

In the framework of the nonlinear Λ-model we investigate propagation of a slow-light soliton in atomic vapours and Bose–Einstein condensates. The velocity of the slow-light soliton is controlled by a time-dependent background field created by a controlling laser. For a fairly arbitrary time dependence of the field we find the dynamics of the slow-light soliton inside the medium. We provide an analytical description for the nonlinear dependence of the velocity of the signal on the controlling field. If the background field is turned off at some moment of time, the signal stops. We find the location and shape of the spatially localized memory bit imprinted into the medium. We show that the process of writing optical information can be described in terms of scattering data for the underlying scattering problem.

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The nonlinear theory of the Lambda-type model of alkali atoms has received a new impetus for further development due to significant progress in the experiments on coherent control of the light–matter interaction. The experiments on hot and cold atomic vapours [1–8] demonstrated intriguing possibilities for realization of nonlinear control over slow-light pulses. From the theoretical point of view, the most important problem is to describe the processes of storing and reading of the optical information. These processes are facilitated by the interaction of light with the medium and are typically controlled via some classical external field. In the linear regime the classical field is assumed to be stronger than the localized pulses of light carrying the information. However, with modern experimental developments it is now evident that for an adequate description of manipulating, storing and reading an optical signal a nonlinear description becomes necessary. In this paper we develop a general nonlinear theory of the control over the dynamics of a slow-light soliton in atomic vapours. In contrast to the linear EIT theory, in our nonlinear approach the controlling field is allowed to change in time in a quite arbitrary way and even to vanish.

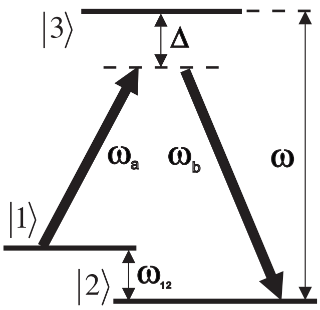

Figure 1. The Λ-scheme for working energy levels of sodium atoms.

The working energy levels of alkali atoms are well approximated by the three-level Λ-scheme. The structure of levels is given in figure 1, where Δ is the detuning of the carrying frequency from the resonance. The medium is described by the 3×3 density matrix ρ in the interaction picture. In order to cancel residual Doppler broadening, two optical beams are chosen to be co-propagating. The electromagnetic fields are described by the Rabi-frequencies $\Omega_{a,b}$. The field Ω_a corresponds to σ^- polarization, while the second Ω_b field corresponds to σ^+ polarization. Within the slowly varying amplitude and phase approximation (SVEPA), dynamics of the atom-field system is well approximated by the Maxwell–Bloch equations [9, 10]. Introducing new variables $\zeta = (x - x_0)/c$, $\tau = t - (x - x_0)/c$ we can rewrite the system of equations in the following matrix form:

$$\partial_\zeta H_I = \mathrm{i}\frac{\nu_0}{4}[D, \rho], \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad (1)$$

$$\partial_\tau \rho = \mathrm{i}\left[\frac{\Delta}{2}D - H_I, \rho\right].$$

The parameter ν_0 is the coupling constant. The matrix $H_I = -\frac{1}{2}(\Omega_a|3\rangle\langle 1| + \Omega_b|3\rangle\langle 2|) + h.c.$ represents the interaction Hamiltonian.

The system of equations (1) is exactly solvable in the framework of the inverse scattering (IS) method [9–12]. This means that the system of equations (1) constitutes a compatibility condition for a certain linear system, namely

$$\partial_\tau \Psi = U(\lambda)\Psi = \frac{\mathrm{i}}{2}\lambda D\Psi - \mathrm{i}H_I\Psi, \qquad (2)$$

$$\partial_\zeta \Psi = V(\lambda)\Psi = \frac{\mathrm{i}}{2}\frac{\nu_0\rho}{\lambda - \Delta}\Psi. \qquad (3)$$

Here, $\lambda \in \mathbb{C}$ is the spectral parameter.

We first describe the state of the physical system before the soliton has entered the medium. In the absence of the soliton the atoms are assumed to be in the state $|1\rangle$. Note that this state is a dark state for the controlling field $\Omega(\tau)$, which means that the atoms do not interact with the field $\Omega(\tau)$ created by the auxiliary laser. We thus build a single-soliton solution on the background of the following state of the overall atom-field system:

$$\Omega_a = 0, \qquad \Omega_b = \Omega(\tau), \qquad |\psi_{at}\rangle = |1\rangle. \qquad (4)$$

This configuration corresponds to a typical experimental setup (see, e.g., [2, 3, 5]).

The state equation (4) satisfies the Maxwell–Bloch equations (1). Using the methods of our previous works [10, 13], we construct the single-soliton solution corresponding to the background field $\Omega(\tau)$ in the form

$$\begin{aligned}\Omega_a &= \frac{(\lambda^* - \lambda)w(\tau,\lambda)}{\sqrt{1+|w(\tau,\lambda)|^2}}\,\mathrm{e}^{\mathrm{i}\theta_s}\,\mathrm{sech}\,\phi_s,\\ \Omega_b &= \frac{(\lambda - \lambda^*)w(\tau,\lambda)}{1+|w(\tau,\lambda)|^2}\,\mathrm{e}^{\phi_s}\,\mathrm{sech}\,\phi_s - \Omega(\tau),\end{aligned} \tag{5}$$

with the atomic state $\rho = |\psi_{at}\rangle\langle\psi_{at}|$, where

$$|\psi_{at}\rangle = \frac{\mathrm{Re}\,\lambda - \Delta - \mathrm{i}\,\mathrm{Im}\,\lambda\tanh\phi_s}{|\lambda-\Delta|}|1\rangle + \frac{\Omega_a}{2|\lambda-\Delta|w(\tau,\lambda)}|2\rangle - \frac{\Omega_a}{2|\lambda-\Delta|}|3\rangle. \tag{6}$$

Here,

$$\begin{aligned}\phi_s &= \frac{\nu_0\zeta}{2}\,\mathrm{Im}\,\frac{1}{\lambda-\Delta} + \mathrm{Re}(z(\tau,\lambda)) + \ln\sqrt{\frac{1+|w(\tau,\lambda)|^2}{1+|w(0,\lambda)|^2}},\\ \theta_s &= -\frac{\nu_0\zeta}{2}\,\mathrm{Re}\,\frac{1}{\lambda-\Delta} + \mathrm{Im}(z(\tau,\lambda)),\end{aligned}$$

while the functions $w(\tau,\lambda)$ and $z(\tau,\lambda)$ are defined below.

In this report we envisage the following dynamics scenario. We assume that the slow-light soliton is propagating in nonlinear superposition with the background field, which is constant at $\tau\to-\infty$ and vanishes at $\tau\to+\infty$. The speed of the slow-light soliton is controlled by the intensity of the background field. Therefore, when the background field decreases, the slow-light soliton slows down and stops, eventually disappearing and leaving behind a standing localized polarization flip, i.e. optical memory bit. Should the background field increase, the soliton will emerge again and accelerate accordingly.

To be specific, we define the asymptotic behaviour for the field $\Omega(\tau)$ in the form

$$\Omega(\tau\to-\infty) = \Omega_0, \qquad \Omega(\tau\to+\infty) = 0. \tag{7}$$

The asymptotic boundary conditions equation (7) dictate the following asymptotic behaviour for the functions $w(\tau,\lambda)$ and $z(\tau,\lambda)$:

$$w(-\infty,\lambda) = w_0 = \frac{\Omega_0}{2k(\lambda)}, \qquad w(+\infty,\lambda) = 0, \tag{8}$$

$$z(-\infty,\lambda) = z_0\tau = \mathrm{i}\frac{|\Omega_0|^2}{4k(\lambda)}\tau, \tag{9}$$

where $k(\lambda) = (\lambda+\sqrt{\lambda^2+|\Omega_0|^2})/2$. The function $z(\tau,\lambda)$ satisfying the asymptotical conditions equation (9) reads

$$z(\tau,\lambda) = z_0\tau + \int_{-\infty}^{\infty}\left(\frac{\mathrm{i}}{2}\Omega^*(\tau')w(\tau',\lambda) - z_0\right)\Theta(\tau-\tau')\,\mathrm{d}\tau'.$$

The function $w(\tau,\lambda)$ is defined by the following relations:

$$w(\tau,\lambda) = \mathrm{i}\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}k(\tau-s)}\Theta(\tau-s)\tilde{w}(s,\lambda)\,\mathrm{d}s, \tag{10}$$

$$\tilde{w}(\tau,\lambda) = \frac{\Omega(\tau)}{2} + \frac{1}{k^2}\left(\frac{|\Omega_0|^2}{4}kw - \frac{\Omega^*(\tau)}{2}(kw)^2\right). \tag{11}$$

Here $\Theta(\tau)$ is the Heaviside step function. We rewrite the relations equations (10), (11) in the form of nonlinear integral equation, namely

$$\tilde{w}(\tau,\lambda) = \frac{\Omega(\tau)}{2} + \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}k(\tau-s)}\Theta(\tau-s)\tilde{w}(s,\lambda)\,\mathrm{d}s \times \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}k(\tau-s)}\Theta(\tau-s)\left(\frac{\Omega^*(\tau)}{2}\tilde{w}(s,\lambda) - \frac{|\Omega_0|^2}{4}\right)\mathrm{d}s. \tag{12}$$

Hence, we can construct a solution $\tilde{w}(\tau,\lambda)$ iterating equation (12) and starting iterations from $\tilde{w}_0(\tau,\lambda) = \frac{1}{2}\Omega(\tau)$.

Note that the last term in equation (11) provides a correction of order k^{-2}, because the function $w(\tau,\lambda)$ asymptotically behaves as $1/k$. It is plain to see that for the constant controlling field this term vanishes. Assuming the real constant background field Ω_0, the imaginary spectral parameter $\lambda_0 = -\mathrm{i}\varepsilon_0$ in the lower half-plane, and in the simplifying approximation $\varepsilon_0 \gg \Omega_0$, the solution equation (5) immediately reduces to the conventional form of the slow-light soliton [10], namely

$$\Omega_a = -\Omega_0\,\mathrm{e}^{\mathrm{i}\theta_{s0}}\,\mathrm{sech}(\phi_{s0}), \qquad \Omega_b = \Omega_0\,\tanh(\phi_{s0}), \tag{13}$$

where

$$\phi_{s0} = \frac{\nu_0\zeta}{2}\,\mathrm{Im}\,\frac{1}{\lambda-\Delta} + \tau\,\mathrm{Re}(z_0), \qquad \theta_{s0} = -\frac{\nu_0\zeta}{2}\,\mathrm{Re}\,\frac{1}{\lambda-\Delta} + \tau\,\mathrm{Im}(z_0), \tag{14}$$

and $z_0 \approx -|\Omega_0|^2/(4\varepsilon_0)$.

It is interesting to note that preserving only the lowest order term with respect to k we obtain from equation (11)

$$w(\tau,\lambda) \approx \frac{\Omega(\tau)}{2k}. \tag{15}$$

Hence, in the lowest order in k we find

$$z(\tau,\lambda) \approx z_0\tau + \int_{-\infty}^{\infty}\left(\frac{\mathrm{i}}{4k}|\Omega(\tau')|^2 - z_0\right)\Theta(\tau-\tau')\,\mathrm{d}\tau'. \tag{16}$$

As one can easily observe, this expression is in agreement with asymptotic condition equation (9). Equations (15), (16) represent, in fact, an approximation of *effective time*, which was used before [5, 14–16] for descriptions of linear or adiabatic regimes of pulse propagation on a time-dependent background.

For an arbitrary dependence of the background field on the retarded time τ, the speed of the slow-light soliton can be represented in the following form:

$$\frac{v_g}{c} = \frac{\partial_\tau\phi_s}{\partial_\tau\phi_s - \partial_\zeta\phi_s}. \tag{17}$$

It can be readily seen that

$$\frac{\partial\phi_s}{\partial\tau} = \frac{\mathrm{Im}(\lambda)|w(\tau,\lambda)|^2}{1+|w(\tau,\lambda)|^2}, \qquad \frac{\partial\phi_s}{\partial\zeta} = \frac{\nu_0}{2}\,\mathrm{Im}\,\frac{1}{\lambda-\Delta}. \tag{18}$$

We have thus found a general solution for the velocity v_g of the slow-light soliton propagating on an arbitrary time-dependent background field in terms of the function $\tilde{w}(\tau,\lambda)$ given by equation (12). This result provides a new way to study dynamics of localized optical signals in the nonlinear EIT systems. It allows us to easily suggest different schemes to slow down, stop, and reaccelerate slow-light solitonic contribution in the probing pulse. With such techniques one can introduce a concept of probing different regions of the media by changing the time that the soliton dwells around a particular location. This time is important in the problems when the interaction between light and some impurities inside the EIT medium is weak and requires slowing the signal down in the vicinity of these impurities in order to gain more information about the structure of the medium.

We also introduce a notion of the distance $\mathcal{L}[\Omega]$ that the slow-light soliton will propagate until it fully stops. This quantity is important because it describes the location of an imprinted memory bit. The brackets $[\cdot]$ indicate a functional dependence of the distance on the controlling field $\Omega(\tau)$. To begin with we consider the case when the field is instantly switched off at the moment $\tau = 0$, i.e. $\Omega(\tau) = \Omega_0\Theta(-\tau)$. Then we easily find the solution for w and z:

$$w(\tau, \lambda) = w_0(\Theta(-\tau) + \Theta(\tau)\,\mathrm{e}^{-\mathrm{i}\lambda\tau}), \qquad z(\tau, \lambda) = z_0\Theta(-\tau)\tau.$$

Hence, we can obtain the distance $\mathcal{L}_0$ that the soliton will propagate through from the moment $\tau = 0$ until its full stop at $\tau \to \infty$:

$$\mathcal{L}_0 = \frac{c|\Delta - \lambda|^2}{\nu_0|\mathrm{Im}(\lambda)|}\ln(1 + |w_0|^2).$$

Here we make use of the assumption that $\mathrm{Im}(\lambda) < 0$.

Now, we can give the definition of the distance $\mathcal{L}[\Omega]$ for a generic field $\Omega(\tau)$ satisfying the conditions equation (7). It is convenient to define it as a relative distance, namely the difference between the absolute coordinate of the stopped signal at the maximum of the signal and the distance $\mathcal{L}_0$. The relative distance reads

$$\mathcal{L}[\Omega] = \frac{2c|\Delta - \lambda|^2}{\nu_0\,\mathrm{Im}(\lambda)}\int_{-\infty}^{\infty}\mathrm{Re}\left(\frac{\mathrm{i}}{2}\Omega^*(\tau)w(\tau, \lambda) - z_0\Theta(-\tau)\right)\mathrm{d}\tau. \tag{19}$$

Using the representation equation (11) we find

$$\mathcal{L}[\Omega] = \frac{2c|\Delta - \lambda|^2}{\nu_0\,\mathrm{Im}(\lambda)}\mathrm{Re}\left(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{e}^{-\mathrm{i}k(\tau - s)}\Theta(\tau - s)\right.$$
$$\left.\times\left(\frac{|\Omega_0|^2}{4}\Theta(-\tau) - \frac{\Omega^*(\tau)}{2}\tilde{w}(s, \lambda)\right)\mathrm{d}s\,\mathrm{d}\tau\right). \tag{20}$$

If we assume that $\Omega(\tau)$ is a smooth function and substitute the solution for $\tilde{w}(\tau, \lambda)$, we find the result in the form of a series

$$\mathcal{L}[\Omega] = \frac{2c|\Delta - \lambda|^2}{2\nu_0\,\mathrm{Im}(\lambda)}\mathrm{Im}\left(\sum_{n=1}^{\infty}\frac{I_n}{k^n}\right),$$

where $I_n[\Omega]$ are regularized Zakharov–Shabat functionals [12]. The first two functionals read $I_1[\Omega] = -\int_{-\infty}^{\infty}(|\Omega(\tau)|^2 - |\Omega_0|^2\Theta(-\tau))\,\mathrm{d}\tau$, $I_2[\Omega] = \frac{1}{2\mathrm{i}}\int_{-\infty}^{\infty}(\Omega^*(s)\partial_s\Omega(s) - \Omega(s)\partial_s\Omega^*(s))\,\mathrm{d}s$. The other functionals can be obtained through the iteration procedure described above. As is usual for the boundary conditions of the finite density type, I_1 is not a proper functional on the complex manifold of physical observables, in the sense described in [12]. In that sense all other functionals in the expansion with respect to k are proper. It is a plausible conjecture that the minimum of the functional of length, equation (20), i.e. $\delta\mathcal{L}[\Omega]/\delta\Omega = 0$ with $\delta^2\mathcal{L}[\Omega]/\delta\Omega^2 > 0$, is achieved when the controlling field is switched off instantly. Therefore it seems intuitively correct that the minimum is delivered by the function $\Omega_0\Theta(-\tau)$ discussed above. This conjecture is also supported by a physically relevant case discussed in our work [13]. In that reference we solve exactly the case when the controlling field vanishes exponentially, i.e. $\Omega(\tau) = \Omega_0(\Theta(-\tau) + \Theta(\tau)\,\mathrm{e}^{-\alpha\tau})$ with $\Theta(0) = \frac{1}{2}$. In this case the minimum of length is delivered by a singular limit $\alpha \to \infty$, i.e. in the regime of instant switching off the controlling field.

Another important characteristic of the system is the shape of the imprinted signal. It is easy to show that the width $\mathcal{W}_0$ of the imprinted memory bit is not at all sensitive to the functional form of $\Omega(\tau)$. This width reads

$$\mathcal{W}_0 = 4c\ln(2+\sqrt{3})\frac{|\Delta-\lambda|^2}{\nu_0|\mathrm{Im}(\lambda)|}. \tag{21}$$

In other words, this exact result is valid regardless of how rapidly we switch the background field off. This means that specification of $\Omega(\tau)$ only influences the location of the stored signal and does not influence its shape. This result is strongly supported by recent experiments [7]. This reference emphasizes the phenomenological fact that the quality of the storage is not sensitive to the regime of switching off the control laser. Our exact result equation (21) provides a rigorous prove for this experimental observation.

Discussion. In this letter we have investigated a mechanism of dynamical control of the slow-light soliton whose group velocity explicitly depends on the background field. For quite general background field, we found the location and shape of the memory bit written into the medium upon stopping the signal. Remarkably, the width of this spatially localized standing polarization flip is not sensitive at all to the functional form of the controlling field and is defined by the parameters of the soliton only. Our results provide us with a motivation to formulate a concept of a *solitonic probing tool*. Indeed, the mechanism of control discussed in this letter allows slowing down the soliton at exact locations inside the medium in a completely controllable fashion. Thus, we can increase the interaction time in the atom-field system at a predefined depth of the sample. In principle, this idea should allow investigation of the structure of the medium. For example, media doped with passive impurities can be investigated. Finally, it is worth mentioning that from equation (20) it is quite evident that the location of the recorded memory bit is defined, through the trace formulae, by the scattering data of the spectral problem equation (3) accompanied by the boundary conditions equation (7). A detailed investigation of this problem will be reported in a forthcoming publication.

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References

[1] Harris S E 1997 *Phys. Today* **50** 36
[2] Hau L N, Harris S E, Dutton Z and Behroozi C H 1999 *Lett. Nature* **397** 594
[3] Liu C, Dutton Z, Behroozi C H and Hau L V 2001 *Lett. Nature* **409** 490
[4] Phillips D F, Fleischhauer A, Mair A, Walsworth R L and Lukin M D 2001 *Phys. Rev. Lett.* **86** 783
[5] Bajcsy M, Zibrov A S and Lukin M D 2003 *Lett. Nature* **426** 638
[6] Braje D A, Balic V, Yin G Y and Harris S E 2003 *Phys. Rev.* A **68** 041801(R)
[7] Dutton Z and Hau L V 2004 *Phys. Rev.* A **70** 053831
[8] Mikhailov E E, Sautenkov V A, Rostovtsev Y V and Welch G R 2004 *J. Opt. Soc. Am.* B **21** 425
[9] Byrne J A, Gabitov I R and Kovačič G 2003 *Physica* D **186** 69
[10] Rybin A V and Vadeiko I P 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 416
[11] Park Q-H and Shin H J 1998 *Phys. Rev.* A **57** 4643
[12] Faddeev L D and Takhtadjan L A 1987 *Hamiltonian Methods in the Theory of Solitons* (Berlin: Springer)
[13] Rybin A V, Vadeiko I P and Bishop A R 2005 *J. Phys. A: Math. Gen.* **38** L177
[14] Grobe R, Hioe F T and Eberly J H 1994 *Phys. Rev. Lett.* **73** 3183
[15] Dey T N and Agarwal G S 2003 *Phys. Rev.* A **67** 033813
[16] Rybin A V, Vadeiko I P and Bishop A R 2005 *Preprint* quant-ph/0504096